

Running of Low-Energy Neutrino Masses, Mixing Angles and CP Violation

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Abstract

We calculate the running of low-energy neutrino parameters from the bottom up, parameterizing the unknown seesaw parameters in terms of the dominance matrix R . We find significant running only if the R matrix is non-trivial and the light-neutrino masses are moderately degenerate. If the light-neutrino masses are very hierarchical, the quark-lepton complementarity relation $\theta_c + \theta_{12} = \pi/4$ is quite stable, but $\theta_{13,23}$ may run beyond their likely future experimental errors. The running of the oscillation phase δ is enhanced by the smallness of θ_{13} , and jumps in the mixing angles occur in cases where the light-neutrino mass eigenstates cross.

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1 Introduction

Low-energy neutrino experiments [1] are providing crucial insight into lepton masses and mixing, though this is still limited in its scope. The most economical model for light neutrino masses is the seesaw model [2], but even the minimal model with three heavy singlet neutrinos contains a total of 18 parameters in the neutrino sector [3, 4]. So far, neutrino experiments provide us with measurements of only four of these [5]: two squared-mass differences Δm_{12}^2 , Δm_{23}^2 , and two neutrino mixing angles $\theta_{23,12}$. There are prospects for measuring one more mixing angle, θ_{13} and the CP-violating Maki-Nakagawa-Sakata phase [6] δ , as well, perhaps, as the overall neutrino mass scale in cosmological data [7] and one combination of Majorana mass parameters in neutrinoless double- β decay [8]. However, even these measurements would fall short of providing complete information on the full set of nine parameters that are in principle observable in low-energy neutrino experiments [9], out of the full total of 18.

Nevertheless, the information available from low-energy neutrino experiments is already striking [5]. The atmospheric mixing angle θ_{23} is close to maximal: $\sin^2 2\theta_{23} = 1.02 \pm 0.04$, and the solar mixing angle θ_{12} is quite large: $\tan^2 \theta_{12} = 0.45 \pm 0.05$. It therefore seems that neutrino mixing must be qualitatively different from the smaller mixing visible between the left-handed quarks, where the largest mixing angle is the original Cabibbo angle: $\sin \theta_C = 0.22$. Such a difference in the quark and neutrino mixing patterns was not widely expected before the experiments, and has given rise to much theoretical discussion and speculation [10, 11, 12].

One of the problematic issues in the interpretation of the low-energy neutrino data is the running of neutrino masses and mixing parameters below and above the seesaw mass scales. This renormalization inevitably introduces some ‘fuzziness’ in the comparison between low-energy measurements and any specific Ansatz for the mass matrix at the seesaw scale, since the renormalization depends on many of the unknown parameters in the seesaw model. The renormalization group equations (RGEs) have been used to study this running extensively, both numerically and analytically ¹. As a result, the observed low-energy neutrino mixing can be obtained starting from either a bimaximal [14] or from an almost diagonal [15] neutrino mass matrix at the Grand Unification (GUT) scale M_{GUT} . In such a situation, understanding the systematical features of the running of neutrino parameters becomes crucial for the interpretation of the neutrino data and for the building of flavour models.

The purpose of this paper is to study comprehensively the dependence of neutrino renormalization effects on *all* the seesaw parameters, paying special attention to obtaining the correct low-energy neutrino measurements. The running of the effective neutrino mass matrix below the lightest singlet neutrino mass is generally well under control and large renormalization effects can be expected only in the case of degenerate light neutrino masses and, in supersymmetric models, for very large values of $\tan \beta$ [16, 17]. However, understanding the renormalization effects above and between the heavy neutrino scales [18, 13] is much more complicated, since new non-trivial dependences on the heavy neutrino Yukawa couplings Y_ν^{ij} are introduced. Because the flavour structure of the new contribution to

¹An up-to-date list of references on this very extensive subject can be found in [13].

the RGEs can be very different from that due to the effective neutrino mass matrix, large effects are possible. Since the couplings Y_ν^{ij} are largely unknown, a typical *top-down* approach taken in previous studies has been to fix the neutrino parameters at M_{GUT} at some chosen values, then to run them down to the electroweak scale and demonstrate that, for this particular choice, the low-energy neutrino mass matrix is compatible with experimental data.

In this paper we take a *bottom-up* approach in which we first fix the known low-energy neutrino parameters to their measured values, and evaluate renormalization towards higher scales consistently in the framework of the minimal supersymmetric seesaw model. In our approach, every set of studied neutrino parameters is physical by construction. We parameterize the nine high-energy parameters of the seesaw mechanism using the orthogonal complex matrix R [19], and scan over all the 18 seesaw parameters by generating the unknown parameters (including phases) randomly. We run the neutrino parameters up to the GUT scale and study the dependence of the renormalization effects on (i) the other observable low-energy and (ii) the high-energy parameters.

We find that significant renormalization effects can occur only when some of the light neutrino masses get *comparable* contributions from two or three heavy neutrinos N_j and/or the light neutrino mass scale is at least moderately degenerate. Because the matrix R_{ij} , known as the dominance matrix [20], characterizes the contributions of the heavy neutrino N_j to the light neutrino masses ν_i , this parametrization turns out to be quite useful for the present study. It has been stated in the literature that the solar angle θ_{12} usually runs more than $\theta_{13,23}$. We find that, for light neutrino masses with a strong normal hierarchy, exactly the opposite occurs. The quark-lepton complementarity relation [12, 21, 22, 23]

$$\theta_C + \theta_{12} = \frac{\pi}{4}, \quad (1)$$

turns out to be very stable while, at the same time, the neutrino angles θ_{13} and θ_{23} may receive renormalization effects larger than the accuracy of plausible future experimental tests. The renormalization of the low-energy oscillation phase δ is generally enhanced compared with the running of mixing angles. Nevertheless, a non-diagonal R -matrix is needed for a large effect also in this case. An interesting feature is the possible crossing of light-neutrino mass eigenstates, which is accompanied by discrete changes in the neutrino mixing pattern, and is correlated with the R -matrix parameters.

Our paper is organized as follows. In Section 2 we present calculational details of our study. In Section 3 we present and discuss our results. Finally, some conclusions are drawn in Section 4.

2 Running of Neutrino Parameters in the MSSM

The superpotential of the minimal supersymmetric standard model (MSSM) with singlet (right-handed) heavy neutrinos is given by

$$W = D^c Y_d Q H_1 + U^c Y_u Q H_2 + E^c Y_e L H_1 + N^c Y_\nu L H_2 + \frac{1}{2} N^c M N^c, \quad (2)$$

where the Yukawa matrices Y are general complex 3×3 matrices and the 3×3 heavy neutrino mass matrix M is symmetric. The Yukawa matrices can be diagonalized by bi-unitary transformations $Y^D = U^\dagger Y V$, where V, U refer to the rotation of the left- and right-chiral fields, respectively. In the case of the symmetric matrix M , $U = V^*$. To explain the neutrino data naturally, the hierarchy in M should preferably be of the same order as the square of the hierarchy in Y_ν [24]. We therefore assume hierarchical heavy-neutrino masses: $M_1 \leq M_2 \leq M_3$.

Integrating out all the heavy singlet neutrinos, one gets the usual dimension-5 effective operator

$$\mathcal{L} = -\frac{1}{2}\kappa LLH_2H_2, \quad (3)$$

which after electroweak symmetry breaking gives masses to the light neutrinos:

$$m_\nu(\mu) = \kappa(\mu)v^2 \sin^2 \beta, \quad (4)$$

where μ is the renormalization scale, $v = 174$ GeV and $\tan \beta = v_2/v_1$ is the ratio of the v.e.v.'s of the corresponding Higgs doublets. Above the heaviest neutrino mass scale, $\mu > M_3$, the light-neutrino mass matrix reads

$$m_\nu(\mu) = Y_\nu^T(\mu)M^{-1}(\mu)Y_\nu(\mu)v^2 \sin^2 \beta. \quad (5)$$

Between the heavy-neutrino mass scales, $M_1 \leq M_2 \leq M_3$, there exist a series of effective theories with, in general, n active heavy neutrinos. The tree-level matching conditions between these theories at the neutrino thresholds are

$$\kappa_{gf}^{(n)}|_{M_n} = \kappa_{gf}^{(n+1)}|_{M_n} + (Y_\nu^{(n+1)})_{ng}M_n^{-1}(Y_\nu^{(n+1)})_{ng}|_{M_n}, \quad (6)$$

where (n) is the number of heavy neutrinos not integrated out. In general, the light-neutrino mass matrix can be written as

$$m_\nu = \left(\kappa^{(n)} + Y_\nu^{T(n)} M^{-1(n)} Y_\nu^{(n)} \right) v^2 \sin^2 \beta. \quad (7)$$

Since m_ν and $Y_e^\dagger Y_e$ can be diagonalized with the unitary matrices V_ν and V_e , respectively, the mixing matrix observable in the low-energy experiments is

$$V_{MNS} = V_e^\dagger V_\nu. \quad (8)$$

While m_ν contains 9 physical parameters, Y_ν and M together contain 18 parameters. The missing 9 parameters crucially affect the physical observables. These include, for example, renormalization-induced lepton-number-violating processes [25, 26, 19, 4, 27] and electric dipole moments [4, 28] in the supersymmetric seesaw model, as well as the renormalization of the light-neutrino parameters [13]. Therefore, to study the dependence of the renormalization of eq. (7) on Y_ν , we parametrize Y_ν with the complex orthogonal matrix R [19]. In the basis in which M is diagonal, we write

$$Y_\nu^{(n)} = (M^D)^{\frac{1}{2}} R^{(n)} (m_\nu^D)^{\frac{1}{2}} V_\nu^\dagger (v \sin \beta)^{-1}, \quad (9)$$

where the matrix R is parametrized in terms of three *complex* angles θ_{12}^R , θ_{13}^R and θ_{23}^R :

$$R = \begin{pmatrix} c_{12}^R c_{13}^R & s_{12}^R c_{13}^R & s_{13}^R \\ -c_{23}^R s_{12}^R - s_{23}^R s_{13}^R c_{12}^R & c_{23}^R c_{12}^R - s_{23}^R s_{13}^R s_{12}^R & s_{23}^R c_{13}^R \\ s_{23}^R s_{12}^R - c_{23}^R s_{13}^R c_{12}^R & -s_{23}^R c_{12}^R - c_{23}^R s_{13}^R s_{12}^R & c_{23}^R c_{13}^R \end{pmatrix}, \quad (10)$$

where $s_{ij}^R \equiv \sin \theta_{ij}^R$ and $c_{ij}^R \equiv \cos \theta_{ij}^R$. Since Y_ν and M are renormalized, obviously also $\overset{(n)}{R}$, which consists of n rows, runs with energy. The RGEs for Y_ν and M can be found in [26], and the RGEs for R were calculated in [29]. Using these, $\overset{(n)}{R}$ has to be evaluated at every heavy neutrino threshold when the matching is performed.

The scale dependence of the effective/combined quantities in (7) is characterized by the differential equation [30, 13]

$$16\pi^2 \frac{d\overset{(n)}{X}}{dt} = (Y_e^\dagger Y_e + \overset{(n)}{Y}_\nu^\dagger \overset{(n)}{Y}_\nu)^T \overset{(n)}{X} + \overset{(n)}{X} (Y_e^\dagger Y_e + \overset{(n)}{Y}_\nu^\dagger \overset{(n)}{Y}_\nu)^T + (2\text{Tr}(\overset{(n)}{Y}_\nu^\dagger \overset{(n)}{Y}_\nu) + 3Y_u^\dagger Y_u) - 6/5g_1^2 - 6g_2^2) \overset{(n)}{X}, \quad (11)$$

where $X = \kappa, Y_\nu^T M^{-1} Y_\nu$. Notice that below the M_1 scale $\overset{(n)}{Y}_\nu = 0$. Therefore one expects large renormalization effects to occur above the heavy-neutrino thresholds for two reasons. First, the Yukawa couplings Y_ν can be large. Secondly, the flavour structure of $Y_\nu^\dagger Y_\nu$ can be very different from the flavour structure of $Y_e^\dagger Y_e$ and κ . Both those effects can be traced back to the values of R via eq. (9). Approximate analytical solutions to eq. (11) have been derived in [13], which allow one to understand the generic behaviour of the renormalization effects. However, due to enhancement/suppression factors and possible cancellations, the exact numerical solutions may differ considerably from those estimates.

3 Results for Normally-Ordered Light Neutrinos

In this Section we present the results of our study for the case of normally-ordered light-neutrino masses, using the following strategy. We start at M_Z , where we fix the measured light-neutrino parameters as $\Delta m_{12}^2 = 8. \times 10^{-5} \text{eV}^2$, $\Delta m_{23}^2 = 2.2 \times 10^{-3} \text{eV}^2$, $\tan^2 \theta_{12} = 0.41$, $\sin \theta_{23} = 0.7$ and $\sin \theta_{13} = 0.05$. We then generate randomly the lightest neutrino mass, the heavy neutrino masses, all the low-energy phases and the initial values for the parameters in the R matrix. We run all the relevant quantities up to M_{GUT} using the 1-loop RGEs for the minimal supersymmetric seesaw model [26, 30]. At every heavy-neutrino threshold we perform the tree-level matching according to eq. (6). To calculate the values of $\overset{(n)}{Y}_\nu$ we use the renormalized values of the R and M . At M_{GUT} we calculate the renormalized light-neutrino parameters. We always keep the ordering of the light neutrino masses fixed, $m_1 < m_2 < m_3$ for the normal and $m_3 < m_1 < m_2$ for the inverse ordering. Because of that the physical range for θ_{12} extends up to $\pi/2$.

In order to accentuate the renormalization effects due to the low-energy neutrino parameters and the parameter matrix R , we do not consider degenerate light neutrinos and

we assume an upper limit $m_1 < 0.1$ eV on the lightest neutrino mass. Although the present most stringent limit on the overall light-neutrino mass scale coming from astrophysics and cosmology [31] is a factor of 2 to 3 weaker, such precision can easily be achieved in the future cosmological experiments [7]. We also minimize the renormalization effects of large $\tan\beta$ studied in [17] by working with the relatively small value of $\tan\beta = 5$. Instead, we study how the large values of Y_ν affect the renormalization effects.

3.1 Renormalization of the Mixing Angles

We start by studying the running of the light-neutrino mixing angles. In Fig. 1 we plot the neutrino mixing angles θ_{ij} at M_{GUT} as functions of the lightest neutrino mass $m_1(M_Z)$ for $R(M_Z) = 1$ (left panel) and for randomly generated R (right panel). In all the figures the neutrino mass parameters are presented in units of eV. The mixing angles θ_{12} , θ_{13} and θ_{23} are represented by green (light), blue (dark) and red (medium) dots, respectively. For $R = 1$ the mixing angles practically do not run: only θ_{12} may change a little for light-neutrino masses close to $m_1 = 0.1$ eV. This is because, for moderate degeneracy, the renormalization of θ_{12} is enhanced by a factor $m_1^2/\Delta m_{12}^2$. This effect would have been larger for larger values of $\tan\beta$ and m_1 .

Turning to the results for the randomly-generated values of R , a certain pattern of renormalization effects emerges. If $m_1 > \sqrt{\Delta m_{12}^2}$, very large changes in the mixing angles may occur. Although θ_{12} tends to change most due to the above-mentioned enhancement factor, also θ_{13} and θ_{23} may gain almost any value. We see that the examples of extreme running considered in the literature are due to having at least a moderately degenerate mass spectrum.

An interesting set of points in Fig. 1 are those gathered around $\theta_{12} \sim 60^\circ$ in the region $m_1 > \sqrt{\Delta m_{12}^2}$. Those correspond to the level crossing of two light-neutrino mass eigenvalues m_1 and m_2 due to renormalization. Because by definition $m_1 < m_2$, this causes a discrete jump in the value of θ_{12} . In the standard parametrization of V_{MNS} , and with small θ_{13} , this implies $\sin\theta_{12} \rightarrow \cos\theta'_{12}$ and, consequently, $\theta'_{12} = 90^\circ - \theta_{12}$. As seen in Fig. 1, this effect is smeared by strong running of θ_{12} and also θ_{13} .

In contrast to the previous discussion, if the mass spectrum is strongly hierarchical: $m_1 < \sqrt{\Delta m_{12}^2}$, the solar angle is much more stable than the mixing angles θ_{13} and θ_{23} . The latter may vary through a range of almost 10° , which is more than the expected precision of future experiments. Moreover, the widths of the θ_{13} and θ_{23} bands in Fig. 1 do not depend on the initial values of the angles θ_{ij} . Thus, *discrimination between different flavour models may be possible in principle in the future*, if one takes into account renormalization effects. We also note that, for hierarchical light-neutrino masses, the quark-lepton complementarity relation $\theta_C + \theta_{12} = \pi/4$ would be maintained with high accuracy at every scale, independently of the unobservable neutrino parameters.

We now study the origins of the effects due to R . In the left panels of Figs. 2, 3 and 4 we plot the distributions of the neutrino mixing angles $\theta_{ij}(M_{GUT})$ as functions of $m_1(M_Z)$ for randomly generated complex parameters θ_{12}^R , θ_{13}^R and θ_{23}^R , respectively. In each figure the other parameters in R are set to zero. The same neutrino mixing angles are plotted in

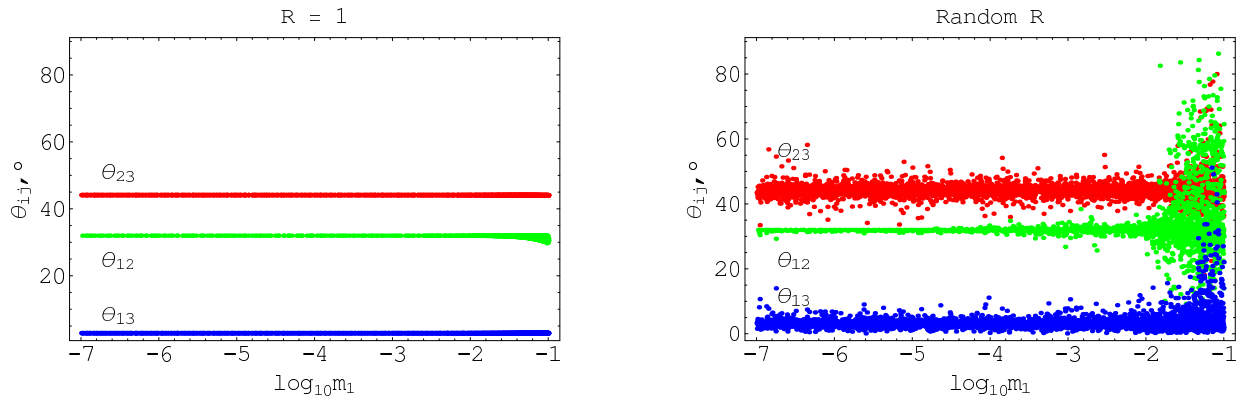


Figure 1: *Neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass at $m_1(M_Z)$ for $R(M_Z) = 1$ (left panel) and randomly generated R (right panel).*

the right panels of Figs. 2, 3, 4 as functions of the absolute values of the corresponding R matrix parameters $|\theta_{12}^R|$, $|\theta_{13}^R|$ and $|\theta_{23}^R|$, respectively.

We see in Fig. 2 that non-zero values of θ_{12}^R affect mostly the renormalization of θ_{12} . A large running effect requires also that the overall light neutrino mass scale be high. On the other hand, non-zero values of θ_{13}^R affect mostly the running of θ_{13} and θ_{23} , as seen in Fig. 3. Again, a relatively high overall light-neutrino mass scale is required for a significant effect. We also learn from Fig. 3 that the level crossing of light mass eigenvalues is induced by non-zero θ_{13}^R , which strongly affects the running of m_1 . The parameter θ_{23}^R affects only the running of θ_{13} and θ_{23} . Fig. 4 shows an interesting feature: in this case the running of θ_{13} and θ_{23} does not depend on m_1 , and significant renormalization effects can be obtained also for very hierarchical light neutrinos.

Comparison of the left and right panels in Figs. 2, 3 and 4 reveals how the renormalization effects depend on the magnitude of the particular parameter θ_{ij}^R . Interestingly, in all the cases the dominant running occurs in the region $|\theta_{ij}^R| \sim \mathcal{O}(1)$. We recall that R can be interpreted to be a dominance matrix [20], *i.e.*, it shows which heavy neutrino contribution dominates in the mass of a particular light neutrino. Therefore our results imply that, in order to have significant renormalization effects, *at least two heavy neutrinos must contribute to one particular light neutrino mass in approximately equal amounts*. If the light neutrino masses are dominated by one heavy neutrino each, no large running is possible unless the light neutrinos are degenerate in mass and $\tan \beta$ is large.

3.2 Renormalization of Masses

The observed hierarchy in the light-neutrino masses, $\sqrt{\Delta m_{12}^2/\Delta m_{23}^2} = 0.18$, is milder than expected in many flavour models. In GUTs with the simplest scalar sector, the neutrino Yukawa couplings are equal to the up-quark Yukawa couplings at M_{GUT} . Contrary to that, phenomenology at the low scale seems to indicate that the neutrino hierarchy is more similar to the less hierarchical down-sector Yukawa couplings, rather than those in the up sector. However, at the moment the lightest neutrino mass and the hierarchy in the heavy-singlet

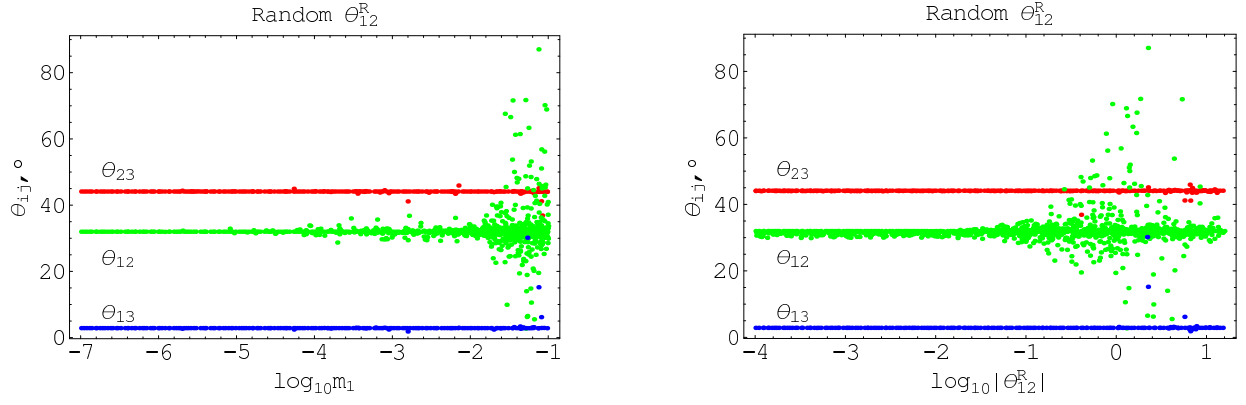


Figure 2: Neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass $m_1(M_Z)$ for random θ_{12}^R (left panel), and as functions of $|\theta_{12}^R|$ (right panel). The rest of the parameters in $R(M_Z)$ vanish.

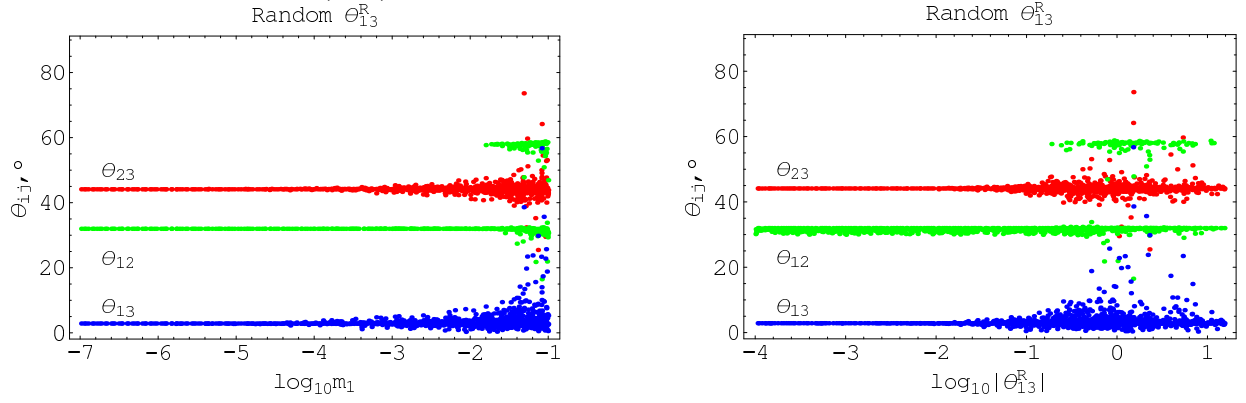


Figure 3: Neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass $m_1(M_Z)$ for random θ_{13}^R (left panel), and as functions of $|\theta_{13}^R|$ (right panel). The rest of the parameters in $R(M_Z)$ vanish.

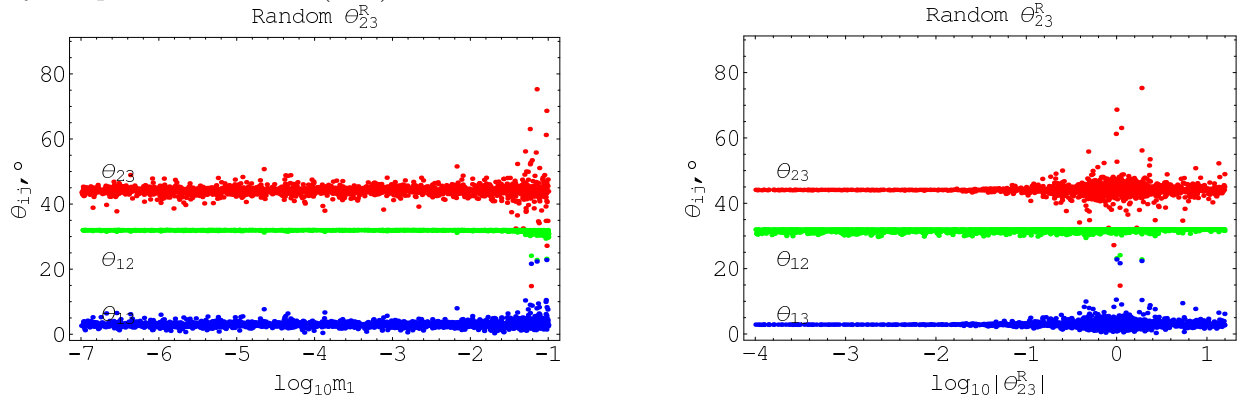


Figure 4: Neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass $m_1(M_Z)$ for random θ_{23}^R (left panel), and as functions of $|\theta_{23}^R|$ (right panel). The rest of the parameters in $R(M_Z)$ vanish.

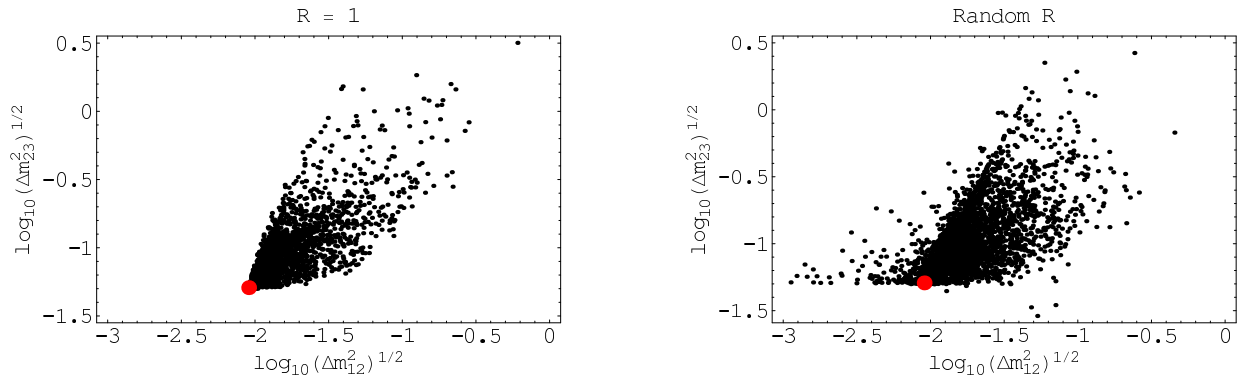


Figure 5: *The distribution of $\sqrt{\Delta m_{23}^2}$ versus $\sqrt{\Delta m_{12}^2}$ at M_{GUT} for $R(M_Z) = 1$ (left panel) and for randomly generated R (right panel).*

sector are unknown, introducing large uncertainties into such considerations. Therefore it is interesting to study also how the masses of the light neutrinos evolve with energy.

In Fig. 5 we plot the distributions of $\sqrt{\Delta m_{12}^2}$ and $\sqrt{\Delta m_{23}^2}$ at M_{GUT} for $R(M_Z) = 1$ (left panel) and for randomly generated R (right panel). The red dot denotes the starting point at M_Z from which value all the other points are generated. *The hierarchy at the GUT scale tends to be larger than at low energies*, although the opposite is also possible for a few points. While for $R = 1$ both mass differences tend to increase, for random R they may also decrease. The abundant points with smaller values of $\sqrt{\Delta m_{12}^2}$ in the right-hand plot correspond to non-zero values of θ_{13}^R . This parameter affects the Yukawa couplings of first generation in such a way that the (12) mass difference may run considerably.

3.3 Renormalization of CP Observables

Our approach in this study is to fix the known neutrino parameters and to vary the unknown ones randomly. At the moment, the only CP-violating observable in the neutrino sector what we have information about is the baryon asymmetry of the Universe, assuming that the cosmological baryon asymmetry is generated via leptogenesis [32]. In this case, it is possible to constrain one combination of the 6 CP-violating phases in the neutrino sector. However, because one can vary the remaining 5 combinations (and also the unknown CP-conserving neutrino parameters), one cannot make any firm prediction for the neutrino parameters [27]. As the predictions for other renormalization-induced CP-violating observables such as the electric dipole moments of the charged leptons are orders of magnitude smaller than the present experimental bound [4, 28], no firm constraints come from this sector either. Although Ref. [17] argues that some systematic renormalization effects in leptogenesis are possible due to the running of the effective light neutrino mass matrix, those are already taken into account in the systematic study of [33].

At the moment, the most realistic possibility seems to be that of measuring the MNS phase δ in future oscillation experiments. The value of this phase, however, is presently unknown. In the left plot of Fig. 6 we present the values of $\delta(M_{GUT})$ against the initial

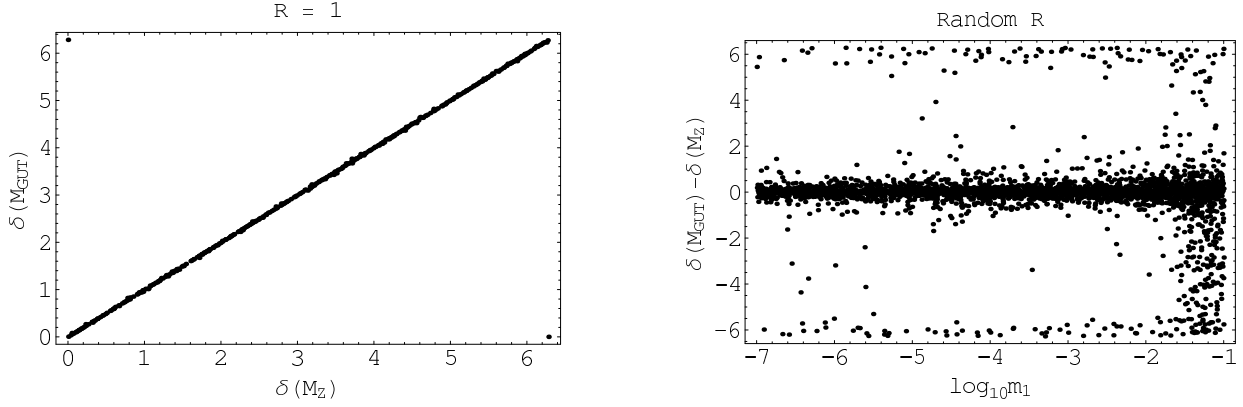


Figure 6: *The MNS phase at high energy $\delta(M_{GUT})$ as a function of its low-energy value $\delta(M_Z)$ for $R(M_Z) = 1$ (left panel), and $\delta(M_{GUT}) - \delta(M_Z)$ as a function of m_1 for randomly generated R (right panel).*

values of the phase for $R(M_Z) = 1$. As seen in the figure there is practically no running of δ in this case. The situation changes considerably for randomly generated R -matrices, as seen in the right panel of Fig. 6 where we plot the change of the phase, $\delta(M_{GUT}) - \delta(M_Z)$, as a function of the lightest neutrino mass m_1 . The running of the MNS phase can be numerically significant and, apart from high values of m_1 , almost independent of the lightest neutrino mass. This behaviour resembles the running of $\theta_{13,23}$ in Fig. 1 and can be traced back to the $1/\theta_{13}$ enhancement of the running of δ [13]².

4 Renormalization Effects for an Inverted Hierarchy of Light Neutrino Masses

We have repeated the earlier analyses also for an inverted hierarchy of light neutrino masses. Because the solar-neutrino mass scale is now higher than the atmospheric one, the Yukawa couplings of the first generations are generally larger. Therefore, all the effects related to the renormalization of the solar parameters are generally enhanced. This can be seen in Fig. 7, where we plot the neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass $m_3(M_Z)$ for $R(M_Z) = 1$ and for randomly-generated R . If $R(M_Z) = 1$, the mixing practically does not run even in the inverted-hierarchy case. The exception is in the high-mass region $m_3(M_Z) \sim 0.1$ eV, when the mass eigenvalues cross and the moderate degeneracy causes discrete jumps in the mixing angles. As expected, for the random choice of R , the angle θ_{12} runs very strongly. The level-crossing stripe around $\theta_{12} \sim 60^\circ$ exists for all values of m_3 , while the angles θ_{13} and θ_{23} run only for large values of m_3 .

²The points around $\pm 2\pi$ in Fig. 6 actually correspond to small running modulo 2π .

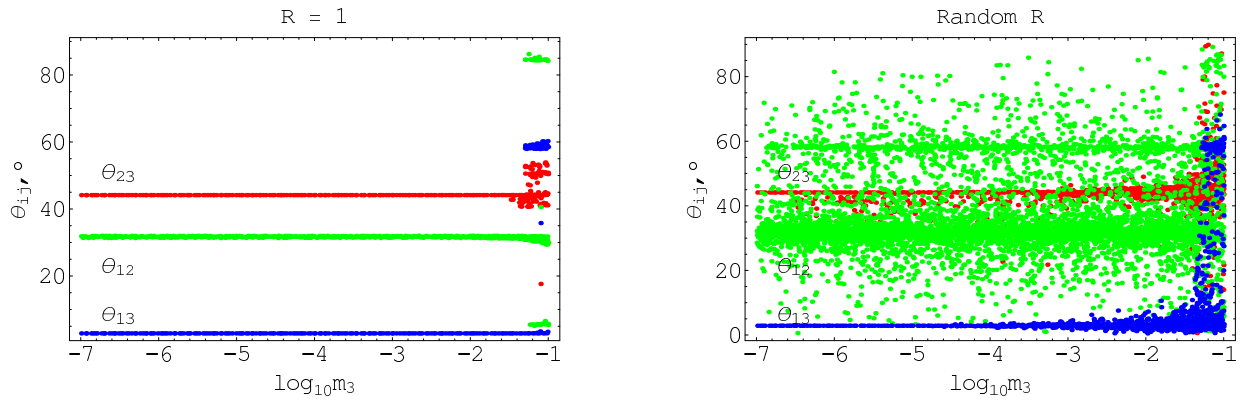


Figure 7: *Neutrino mixing angles at M_{GUT} as functions of the lightest neutrino mass $m_3(M_Z)$ for $R(M_Z) = 1$ (left panel) and for randomly generated R (right panel).*

5 Discussion and Conclusions

We have studied how the RGE running of neutrino mixing depends on the unknown seesaw parameters. We have taken a bottom-up approach in which we fix the known low-energy neutrino parameters to their measured values. Parametrizing the nine high-energy parameters of the seesaw mechanism via the dominance matrix R , we have scanned over all the 18 seesaw parameters by generating the unknown parameters randomly. The fact that the matrix R_{ij} measures the heavy neutrino N_j contribution to the light neutrino ν_i means that this parametrization is particularly valuable for this study. We have compared the results for random R elements with the simple case $R = 1$.

We have found that significant running effects can occur only when some of the light-neutrino masses have comparable contributions from more than one heavy neutrino N_i , and the light-neutrino mass scale is at least moderately degenerate. For a normal hierarchy of neutrino masses, the complementarity relation (1) between neutrino and quark mixing angles evolves very little between the GUT scale and the electroweak scale. However, the other oscillation angles $\theta_{13,23}$ run rather more than θ_{12} and also beyond the expected measurement errors. In certain cases, we observe level crossing in the light-neutrino mass eigenstates that is accompanied by jumps in the oscillation angles. The running of the CP-violating oscillation phase δ is strong for random R but insignificant for $R = 1$.

The analysis presented here complements the top-down approach often adopted elsewhere. It reveals some of the pitfalls in inferring properties of the neutrino mass matrix generated at the GUT scale from current low-energy measurements alone, in the absence of supplementary theoretical or phenomenological input. We hope that these results may serve as useful aids in the attempt to understand the neutrino mass matrix, which has already revealed several surprises. The results presented here demonstrate that our low-energy measurements are far from telling us the whole story.

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